

A note on D-brane Interactions in the IIA Plane-Wave Background

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Abstract

We compute the D-brane tensions in the type IIA plane-wave background by comparing the interaction potential between widely separated D-branes in string theory with the supergravity mode exchange between the D-branes. We found that the D-brane tensions and RR charges in the plane-wave background are the same as those in the flat space. Also we work out the stringy halo behavior[9] of the $DpD\bar{p}$ spacelike branes and find the explicit dependence on the light cone separation r^+ . This suggests that the detailed tachyon dynamics for the spacelike $DpD\bar{p}$ branes are different from those in the flat space case. We also discuss specific features of the exchange amplitudes in relation to the geometric properties of the IIA plane wave background. When branes are located at focal points, full or partial restoration of the translation invariance occurs and the amplitudes are similar to those in the flat space.

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1 Introduction

In a series of papers[1],[2],[3],[4] ,[5],[6],[7], [10] simple type IIA string theory on the plane wave background have been studied, parallel to the development of the IIB string theory on the plane wave background [11] ,[12], [13] ,[14],[15], [16]. The background for the string theory is obtained by compactifying the 11- dimensional plane wave background on a circle and taking the small radius limit. By working out the details of the resulting string theory one can obtain the useful information about the M-theory on the 11-dimensional plane wave background. However the resulting string theory in itself has many nice features and is worthy of the detailed study. It admits light cone gauge where the string theory spectrum is that of the free massive theory, which is similar to the IIB case. The world sheet enjoys (4,4) world sheet supersymmetry and the supersymmetry commutes with the Hamiltonian so that all members of the same supermultiplet has the same mass, which is different from the IIB theory. The various 1/2 BPS D-brane states were analyzed both in the light cone gauge and in the covariant formalism[1],[2]. Finally the boundary state formalism has developed in [2],[7]. And the usual open-closed string channel duality was shown to hold for the supersymmetric D-brane configurations. In this note, we study the interactions of the D-branes and the anti-D-branes in the Type IIA plane wave background based on the boundary state formalism. As applications of this development, we work out the D-brane tensions, Ramond-Ramond (RR) charges and the stringy halo[8, 9] of the D-branes in the IIA plane wave background.

First we work out the open string partition function in the presence of widely separated D-branes. By taking a suitable limit, we find the contribution from the supergravity modes. By comparing this string theory computation with the corresponding supergravity calculation, we find that the supersymmetric D-brane tensions and RR charges are the same as those in the flat space. Similar result was obtained for the various IIB plane wave backgrounds [15]. In fact, we closely follow their approach in this paper. As explained in [15], for the timelike branes, the result is more or less trivial. The interaction between a brane and an anti-brane is given by the overlap $\langle D\bar{p}|\Delta|Dp\rangle$ where Δ is the closed string propagator and $|Dp\rangle, |D\bar{p}\rangle$ are the corresponding boundary states. Due to the boundary condition, they satisfy $\hat{p}^+|Dp\rangle = \hat{p}^+|D\bar{p}\rangle = 0$ where \hat{p}^+ is the light cone momentum operator. This condition projects the closed string propagator to the $\hat{p}^+ = 0$ subspace and these states propagate as in the flat space.

However it is nontrivial to compute the tension of the space-like D-branes. Actual calculation shows that the supersymmetric D-brane tensions and RR charges are the same as those in flat space irrespective of their position in the flat space. In view of this, one might think that space-like D-branes in the IIA plane wave background would behave in the same

way as the timelike branes do. But another calculation shows that this is not the case. We ask at which distance $DpD\bar{p}$ branes would develop the divergence of the amplitude, which indicates the instability associated with the decay into the closed string channel. In the flat space case, $DpD\bar{p}$ branes develop the divergence at the interbrane distance $X_H \equiv \sqrt{2\pi^2\alpha'}$. This was interpreted as D-branes having a stringy halo. This was also discussed in [9] in the context of $D(-1)$ instantons in the IIB plane wave background. We will see that the space-like $DpD\bar{p}$ branes in the IIA plane wave background would develop the stringy halo but the halo itself depends on the mass parameter, which again has the nontrivial dependence on the light-cone separation r^+ . Thus we confirm the different behavior of the stringy halo for the various spacelike D-branes in the IIA plane wave background than that of the spacelike D-branes in the flat space. This was observed for $D(-1)$ brane in the IIB plane wave background and we confirm that spacelike D-branes in the IIA plane wave background have the similar characters. On the other hand, timelike branes in the IIA plane wave background has the same stringy halo behavior as the D-branes in the flat space since in the boundary state formalism the closed string propagator projects to the $p^+ = 0$ subspace. Thus one can say that the timelike branes and the spacelike branes in the IIA plane wave background would behave in the similar way at the large interbrane distance but they behave differently at the short interbrane distance. In view of this, the fact that spacelike branes in the plane wave background have the same tension as that of the D-branes in the flat space is quite nontrivial. Also this would indicate that the spacelike branes are the good probes to study the different behaviors of D-branes in the IIA plane wave background than that of D-branes in the flat space. Thus the next logical step would be to study the various interaction terms of the D-branes using either boundary state formalism or using the Dirac-Born-Infeld type action of the D-branes in the IIA plane wave background. This is also needed to understand the tachyon dynamics in the IIA plane wave background.

Finally we discuss the specific features of the integrated amplitudes. Motivated by the similar work in the Type IIB side [19], we can deduce the many features of the integrated amplitudes from the geometrical properties of the Type IIA plane wave. Several features are traced to the lack of translational invariances. For specific values of r^+ the translational invariances are fully or partially restored, which is due to the focusing of the geodesics. The reinstated translational invariance can be seen from the world sheet calculation as well.

The content of this note is as follows. In section 2, we present the string theory calculation for the interaction potential between two D-branes. Then we discuss the case where the interbrane distance is large so that we can compare with the tensions obtained from the field theory calculation. This is followed by the consideration of the short distance limit where the stringy halo develops. In section 3 we carry out the supergravity analysis. In section 4, followed is the discussion of the integrated amplitudes in relation to the geometric

properties of the Type IIA plane wave background. In the appendix, we provide the relevant information of the supergravity bulk action relevant for the calculation in the text.

2 String Theory Calculation

The IIA plane wave background of our interest is given by

$$ds^2 = -2dX^+dX^- - A(x^I)(dX^+)^2 + \sum_{i=1}^8 dX^I dX^I, \quad (2.1)$$

$$F_{+123} = \mu, F_{+4} = \frac{\mu}{3} \quad (2.2)$$

and

$$A(x^I) = \sum_{i=1}^4 \frac{\mu^2}{9} (X^i)^2 + \sum_{i'=5}^8 \frac{\mu^2}{36} (X^{i'})^2 \quad (2.3)$$

where $X^\pm = \frac{1}{\sqrt{2}}(X^0 \pm X^9)$. With the presence of the Ramond-Ramond background, the above plane wave background has $SO(3) \times SO(4)$ where $SO(3)$ acts on 1,2,3 directions and $SO(4)$ acts on 5,6,7,8 directions. We use the convention that unprimed coordinates denote 1,2,3,4 directions while primed coordinates denote 5,6,7,8 directions. For the worldsheet coordinates we use $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$.

The worldsheet action for the closed string theory is given by

$$\begin{aligned} S_{LC} = & -\frac{1}{4\pi\alpha'} \int d^2\sigma (-\partial_\mu X^+ \partial^\mu X^- + \partial_\mu X^I \partial^\mu X^I + \frac{m^2}{9} \sum_{i=1}^4 X^i X^i + \frac{m^2}{36} \sum_{i'=1}^4 X^{i'} X^{i'}) \\ & + 2 \sum_{b=\pm} (-i\psi_b^1 \partial_+ \psi_b^1 - i\psi_b^2 \partial_- \psi_b^2) + \frac{2im}{3} \psi_+^2 \gamma^4 \psi_-^1 - \frac{im}{3} \psi_-^2 \gamma^4 \psi_+^1 \end{aligned} \quad (2.4)$$

where $m = \alpha' p^+ \mu$ and γ^I are 8×8 matrices satisfying $\{\gamma^I, \gamma^J\} = \delta^{IJ}$. The sign of subscript ψ_\pm^A denotes the eigenvalue of γ^{1234} while the superscript $A = 1, 2$ denotes the eigenvalue of γ^9 . The theory has two supermultiplets $(X^i, \psi_-^1, \psi_+^2)$, $(X^{i'}, \psi_+^1, \psi_-^2)$ of (4,4) worldsheet supersymmetry with the mass $\frac{m}{3}$ and $\frac{m}{6}$ respectively. The fermions of the first supermultiplet have γ^{12349} eigenvalue of 1 while those of the second have the eigenvalue of -1 .

One can also consider the open strings with a suitable boundary conditions at the end of the open strings. One can use the same form of the action (2.4) except for the interval for σ runs from 0 to π while for closed string σ runs from 0 to 2π . Possible supersymmetric boundary conditions or possible supersymmetric D-brane configurations were considered in [1],[2] and [7]. Corresponding boundary states were constructed in [2] and [7]. For the spacelike branes, the possible type of supersymmetric D-brane configurations were tabulated in [2]. D0,D2,D4,D6 spacelike branes are supersymmetric and one has to choose particular Neumann directions to satisfy the supersymmetric conditions. We first carry out the string

theory calculation in the open string channel to get a correctly normalized amplitude and extract the contribution from the lowest closed string modes. We then match these results with the supergravity calculation. As explained in [13], in the standard light cone gauge $X^\pm = \alpha' p^\pm \tau$, X^\pm are automatically Neumann directions. Thus *we will use the nonstandard light cone gauge for the open string* $X^+ = \frac{r^+}{\pi} \sigma$ where r^+ is the brane separation along the X^+ coordinate and σ is the worldsheet coordinate $\sigma \in [0, \pi]$. The Virasoro constraint determines X^- to be a Dirichlet direction as well. In this gauge, the mass parameter appearing in the string action is

$$m = \frac{\mu r^+}{\pi}. \quad (2.5)$$

The interaction energy between the branes can be written as

$$\begin{aligned} E &= 2 \cdot \frac{1}{2} Tr(-1)^{F_s} \ln L_0 \\ &= \int_0^\infty \frac{dt}{t} Tr(-1)^{F_s} e^{-L_0 t} \end{aligned} \quad (2.6)$$

where F_s is the spacetime fermion number and the trace is taken over the open strings stretched between the branes and $L_0 = p_- H_{lc}$ with H_{lc} being the light-cone Hamiltonian.

For $DpDp$ and $DpD\bar{p}$ branes separated in r^+, r^- and the transversal $x^D, x^{D'}$ directions.

$$\begin{aligned} E &= \int_0^\infty \frac{dt}{t} e^{-2\pi t \left(\frac{2f(x_1, x_2)}{4\pi^2 \alpha'} \right)} (2 \sinh \pi t \omega_0)^{2-n_N} (2 \sinh \pi t \omega'_0)^{2-n'_N} \\ &\quad \cdot \frac{f_A^{\omega_0}(q)^4 f_A^{\omega'_0}(q)^4}{f_1^{\omega_0}(q)^4 f_1^{\omega'_0}(q)^4} \end{aligned} \quad (2.7)$$

$$\begin{aligned} f(x_1, x_2) &= i r^+ r^- + \frac{m_1 r^+}{2 \sinh m_1 r^+} [((x_1^{\vec{D}})^2 + (x_2^{\vec{D}})^2) \cosh m_1 r^+ - 2 x_1^{\vec{D}} \cdot x_2^{\vec{D}}] \\ &\quad + \frac{m_2 r^+}{2 \sinh m_2 r^+} [((x_1^{\vec{D}'})^2 + (x_2^{\vec{D}'})^2) \cosh m_2 r^+ - 2 x_1^{\vec{D}'} \cdot x_2^{\vec{D}'}] \end{aligned} \quad (2.8)$$

where $q = e^{-2\pi t}$ and n_N is the number of Neumann directions along 1234 while n'_N is the number of those along 5678 with $n_N + n'_N = p + 1$. Also $m_1 = \frac{\mu}{3}$, $m_2 = \frac{\mu}{6}$, $\omega_0 = \frac{m}{3}$, $\omega'_0 = \frac{m}{6}$ while m is given by (2.5). The i factor in $i r^+ r^-$ of (2.8) appears since we consider the Euclideanized IIA plane wave background[9].

The value $A = 1$ for $DpDp$ and $A = 4$ for $DpD\bar{p}$ configurations.

$$\begin{aligned} f_1^{(m)}(q) &= q^{-\Delta_m} (1 - q^m)^{1/2} \prod_{n=1}^\infty (1 - q^{\sqrt{n^2 + m^2}}) \\ f_4^{(m)}(q) &= q^{-\Delta'_m} \prod_{n=1}^\infty (1 - q^{\sqrt{(n - \frac{1}{2})^2 + m^2}}) \end{aligned} \quad (2.9)$$

while Δ_m, Δ'_m are defined as

$$\begin{aligned}\Delta_m &= -\frac{1}{(2\pi)^2} \sum_{p=1}^{\infty} \int_0^{\infty} ds e^{-p^2 s} e^{-\frac{\pi^2 m^2}{s}} \\ \Delta'_m &= -\frac{1}{(2\pi)^2} \sum_{p=1}^{\infty} (-1)^p \int_0^{\infty} ds e^{-p^2 s} e^{-\frac{\pi^2 m^2}{s}}\end{aligned}\quad (2.10)$$

The modular transformation properties are given by

$$f_1^m(e^{-2\pi t}) = f_1^{mt}(e^{\frac{-2\pi}{t}}), \quad f_4^m(e^{-2\pi t}) = f_2^{mt}(e^{\frac{-2\pi}{t}}) \quad (2.11)$$

where $f_2^{(m)}(q)$ is defined by

$$f_2^{(m)}(q) = q^{-\Delta_m} (1 + q^m)^{1/2} \prod_{n=1}^{\infty} (1 + q^{\sqrt{n^2 + m^2}}).$$

The $DpDp$ and $DpD\bar{p}$ brane configurations are aligned along the special directions which make the resulting Dp-brane configurations supersymmetric. One can see that in (2.7), half of the contribution comes from one supermultiplet with the mass parameter $\frac{m}{3}$ and the other half comes from the other supermultiplet with the mass parameter $\frac{m}{6}$. . The large distance behavior of (2.7) comes from the leading behavior of the integrand for small t . In order to match with the field theory calculation, where the calculation is done on the Lorentzian spacetime, we should take account of the Wick rotation effect. Equivalently one follows the convention of [15, 19] so that one works in the Lorentzian signature for space time with a suitable ϵ prescription, that is, one might start with

$$E = i \int_0^{\infty} \frac{dt}{t} Tr(-1)^{Fs} e^{i(L_0 + i\epsilon)t} \quad (2.12)$$

Either way, one obtains

$$E_{Dp-Dp} = -4\pi(4\pi^2\alpha')^{3-p} \sin^2 \frac{\mu r^+}{3} \sin^2 \frac{\mu r^+}{6} I_0^{9-p} \quad (2.13)$$

$$E_{Dp-D\bar{p}} = -4\pi(4\pi^2\alpha')^{3-p} \cos^2 \frac{\mu r^+}{3} \cos^2 \frac{\mu r^+}{6} I_0^{9-p} \quad (2.14)$$

where I_0^{9-p} is the integrated propagator over $(p+1)$ -longitudinal directions with the separation in r^+, r^- and the transverse directions for the associated propagator $G_0(x_1, x_2)$ satisfying $\square G_0(x_1, x_2) = i\delta(x_1 - x_2)$. This is given by

$$I_0^{9-p} = i \frac{(i\mu r^+)^{3-p}}{2} (2\pi)^{p-4} \tilde{f}(x_1, x_2)^{p-3} \Gamma(3-p) \frac{(\frac{\mu}{3})^{2-n_N} (\frac{\mu}{6})^{2-n'_N}}{\sin^2 \frac{\mu r^+}{3} \sin^2 \frac{\mu r^+}{6}}. \quad (2.15)$$

where $\tilde{f}(x_1, x_2)$ is given by

$$\begin{aligned}\tilde{f}(x_1, x_2) &= r^+ r^- + \frac{m_1 r^+}{2 \sin m_1 r^+} [((x_1^{\vec{D}})^2 + (x_2^{\vec{D}})^2) \cos m_1 r^+ - 2x_1^{\vec{D}} \cdot x_2^{\vec{D}}] \\ &+ \frac{m_2 r^+}{2 \sin m_2 r^+} [((x_1^{\vec{D}'})^2 + (x_2^{\vec{D}'})^2) \cos m_2 r^+ - 2x_1^{\vec{D}'} \cdot x_2^{\vec{D}'}].\end{aligned}\quad (2.16)$$

We compare this expression with that from the field theory calculation on the next section and compute the tension of the D-branes.

Now let us concentrate on the large t behavior of the expression. The integrand without $\frac{1}{t}$ factor is reduced to

$$\exp(-2\pi t[\frac{f(x_1, x_2)}{2\pi^2\alpha'} + \frac{\omega_0}{2}(2 - n_N) + \frac{\omega'_0}{2}(2 - n_{N'}) + 4(-\Delta'_{\omega_0} + \Delta_{\omega_0}) + 4(-\Delta'_{\omega'_0} + \Delta_{\omega'_0})]) \quad (2.17)$$

In the $\mu \rightarrow 0$ limit, the expression inside the bracket is reduced to

$$\frac{1}{2}(\frac{2ir^+r^- + (x_1^{\vec{D}} - x_1^{\vec{D}'})^2 + (x_1^{\vec{D}'} - x_1^{\vec{D}})^2}{2\pi^2\alpha'} - 1) \quad (2.18)$$

which is the same expression as one would obtain for the flat space case. Thus if the interdistance is smaller than $X_H = \sqrt{2\pi^2\alpha'}$ in the flat space the amplitude of Dbrane-anti D brane develops the divergence. This is interpreted as D-branes having a stringy halo. This originates from the fact that the bulk of the open strings which end on them can reach out in the transverse directions, forming a region of potential activity of size set by X_H . This halo means that the D-branes can interact with each other before zero separation, as there is enhancement of the physics of interaction by new light states by the entanglement of halos, and the cross over into the annihilation channel begin before the branes are coincident [9]. If we consider the stringy-halo effect in the IIA plane wave background, we should extract the ground state energy of $DpD\bar{p}$ states, which is given by $\frac{\omega_0}{2}(2 - n_N) + \frac{\omega'_0}{2}(2 - n_{N'})$ where $n_N + n_{N'} = p + 1$. Thus the stringy halo develops when

$$2f(x_1, x_2) = 2\pi^2\alpha'(8(\Delta'_{\omega_0} - \Delta_{\omega_0}) + 8(\Delta'_{\omega'_0} - \Delta_{\omega'_0})) \quad (2.19)$$

where in the $\mu \rightarrow 0$ limit the left hand side is reduced to the interdistance of $DpD\bar{p}$ branes and the right hnd side is reduced to $2\pi^2\alpha'$. Due to the nonstandard light cone gauge we should choose for the spacelike branes, $\omega_0 = \frac{\mu r^+}{3\pi}$ and $\omega'_0 = \frac{\mu r^+}{6\pi}$, the right hand side has explicit dependence on the lightcone separation μr^+ . This calculation suggests that the tachyon dynamics for the spacelike branes in the IIA plane wave background would be different from those in the flat space case. Thus it would be interesting to understand the tachyon potentials for $DpD\bar{p}$ branes in detail. The corresponding study in the flat space case produces huge literatures[17].

3 Field Theory Calculation

We start from the type IIA supergravity action appeared in the appendix with D-brane source terms

$$S_p = -T_p \int d^{p+1}x \sqrt{-\tilde{g}} e^{\frac{p-3}{4}\Phi} + \mu_p \int A_{[p+1]} \quad (3.1)$$

where \tilde{g} stands for the induced metric on the worldvolume while T_p and μ_p are the brane tension and the RR charge of the D-brane in consideration.

In the appendix, we obtain the gravity action to the quadratic order in the fluctuation around the plane wave background using the light-cone gauge.

The resulting action is a sum of decoupled terms characterized by coefficients c and M

$$S_\psi = \frac{1}{M} \int d^{10}x \psi_\alpha^\dagger (\square - 2i\mu c \partial_-) \psi_\alpha. \quad (3.2)$$

In our case, ψ is in a tensor representation of $SO(3) \times SO(4)$ and a contraction of tensor indices is understood. The details are explained in the appendix. The source terms contain the contribution of ψ_α with $\varepsilon = \pm 1$ for brane/anti-brane respectively.

$$S_{\text{source}} = \int d^{10}x \delta^{9-p}(x - x_0) k(\psi_\alpha + \varepsilon \bar{\psi}_\alpha). \quad (3.3)$$

The contribution of such a mode to the interaction is given by

$$E = 2M\epsilon k^2 \cos \mu c r^+ I_0^{9-p}. \quad (3.4)$$

3.1 D0-brane

We take the world volume direction to be $X^i = X^1$. From the above source term (3.1), we have

$$\mathcal{L}_{\text{source}} = \frac{iT_0}{2}(h_{11}^\perp - \frac{3}{2}\phi) \pm \mu_0 a_1. \quad (3.5)$$

Using the notation of the appendix, this can be rewritten as

$$\mathcal{L}_{\text{source}} = \frac{iT_0}{2}(h_{11}^\perp - \frac{1}{2}\phi_0 - \frac{3}{8}(\phi_2 + \bar{\phi}_2) - \frac{1}{8}(\phi_0 + \bar{\phi}_0)) \pm \mu_0 \frac{1}{4\sqrt{2}}(\beta_{21} + \bar{\beta}_{21} + \beta_{41} + \bar{\beta}_{41}) \quad (3.6)$$

where the definitions of the fields are given in the appendix. The value of M, k, c are summarized in the following table

ψ	h_{11}^\perp	ϕ_0	ϕ_2
(M, k, c)	$(4\kappa^2, \frac{iT_0}{4}, 0)$	$(\frac{32}{3}\kappa^2, -\frac{iT_0}{8}, 0)$	$(\frac{64}{9}\kappa^2, -\frac{3iT_0}{16}, \frac{\mu r^+}{3})$

ψ	ϕ_6	β_{21}	β_{41}
(M, k, c)	$(64\kappa^2, -\frac{iT_0}{16}, \mu r^+)$	$(16\kappa^2, \frac{\pm\mu_0}{4\sqrt{2}}, \frac{\mu r^+}{3})$	$(16\kappa^2, \frac{\pm\mu_0}{4\sqrt{2}}, \frac{2\mu r^+}{3})$

The sum of the contribution is given by

$$E = -\kappa^2 I_0^9 \left[T_0^2 + \frac{T_0^2}{2} \cos \mu r^+ + \left(\frac{T_0^2}{2} \mp \mu_0^2 \right) \cos \frac{\mu r^+}{3} \mp \mu_0^2 \cos \frac{2\mu r^+}{3} \right] \quad (3.7)$$

comparing with the string result, we get

$$T_0^2 = \mu_0^2 = \frac{\pi}{\kappa^2}(4\pi^2\alpha')^3. \quad (3.8)$$

Since we have $SO(3) \times SO(4)$ symmetry, the same result would be obtained if D0-brane world volume direction is X^i with $i=1,2,3$. This exhausts all the possible of the supersymmetry D-brane configurations. In the string theory analysis, we have the precisely the same condition for the supersymmetric D0-brane.

3.2 D2 brane

According to the analysis in the string theory[18], the worldvolume directions of the supersymmetric D2-brane can be taken to be (124) or (456) directions. The other configurations are equivalent to either of these by $SO(3) \times SO(4)$ rotations. If we take the world volume direction to be (124),

$$\begin{aligned} \mathcal{L}_{\text{source}} &= \frac{iT_2}{2}(h_{11} + h_{22} + h_{44} - \frac{1}{2}\phi) \pm \mu_2 a_{412} \\ &= \frac{iT_2}{2}(h_{11}^\perp + h_{22}^\perp + \frac{1}{2}\phi_0 - \frac{3}{8}(\phi_2 + \bar{\phi}_2) - \frac{1}{8}(\phi_0 + \bar{\phi}_0)) \\ &\quad \pm \mu_2 \frac{1}{4\sqrt{2}}(\beta_{23} + \bar{\beta}_{23} - \beta_{43} - \bar{\beta}_{43}) \end{aligned} \quad (3.9)$$

Again the contribution of the each field to the interaction energy is given by the table below.

ψ	$h_{11}^\perp, h_{22}^\perp$	ϕ_0	ϕ_2
(M, k, c)	$(4\kappa^2, \frac{iT_2}{4}, 0)$	$(\frac{32}{3}\kappa^2, \frac{iT_2}{8}, 0)$	$(\frac{64}{9}\kappa^2, -\frac{i3T_2}{16}, \frac{\mu r^+}{3})$

ψ	ϕ_6	β_{23}	β_{43}
(M, k, c)	$(64\kappa^2, -\frac{iT_2}{16}, \mu r^+)$	$(16\kappa^2, \frac{\pm\mu_2}{4\sqrt{2}}, \frac{\mu r^+}{3})$	$(16\kappa^2, \frac{\pm\mu_2}{4\sqrt{2}}, \frac{2\mu r^+}{3})$

The interaction energy is given by

$$E = -\kappa^2 I_0^7 \left[T_2^2 + \frac{T_2^2}{2} \cos \mu r^+ + \left(\frac{T_2^2}{2} \mp \mu_2^2 \right) \cos \frac{\mu r^+}{3} \mp \mu_2^2 \cos \frac{2\mu r^+}{3} \right] \quad (3.10)$$

Again comparing with the string theory result, we get

$$T_2^2 = \mu_2^2 = \frac{4\pi^2\alpha'}{\kappa^2} \quad (3.11)$$

When the world volume direction of D2-brane is 456, one can carry out similar computation, which gives the same values of T_2 and μ_2 .

3.3 D4 brane

If we take the world volume direction of the D4-brane to be (35678), then the coupling

$$\mu_4 \int A_{[5]} = \mu_4 \int d^5 x A_{35678} = -\mu_4 \int d^5 x \varepsilon^{124}{}_{35678} A_{124} \quad (3.12)$$

in the light cone gauge where $\varepsilon_{i_1 i_2 \dots i_8}$ is the totally antisymmetric tensor of rank 8. Thus the D4-brane in consideration is magnetically charged to the 3-form potential but for the computational purpose the contribution from (3.12) would be the same as in the D2-brane case if we replace μ_2 by μ_4 . Thus we have to figure out the contribution from other remaining source terms.

$$\begin{aligned} \mathcal{L}'_{\text{source}} &= \frac{iT_4}{2}(h_{33} + h_{55} + h_{66} + h_{77} + h_{88} + \frac{\phi}{2}) \\ &= \frac{iT_4}{2}(h_{33}^\perp + h_{55}^\perp + h_{66}^\perp + h_{77}^\perp + h_{88}^\perp \\ &\quad - \frac{1}{2}\phi_0 + \frac{1}{8}(\phi_6 + \bar{\phi}_6) + \frac{3}{8}(\phi_2 + \bar{\phi}_2)) \end{aligned} \quad (3.13)$$

The contribution of ϕ_0, ϕ_2, ϕ_6 are already worked out in D2-brane case. Working out the contribution from the remaining gravitational sector we obtain the interaction energy

$$E = -\kappa^2 I_0^5 \left[T_4^2 + \frac{T_4^2}{2} \cos \mu r^+ + \left(\frac{T_4^2}{2} \mp \mu_4^2 \right) \cos \frac{\mu r^+}{3} \mp \mu_4^2 \cos \frac{2\mu r^+}{3} \right]. \quad (3.14)$$

By the comparison with the string theory result, we get

$$T_4^2 = \mu_4^2 = \frac{\pi}{\kappa^2} (4\pi^2 \alpha')^{-1}. \quad (3.15)$$

If the world volume direction is (12378), we can work out the similar computation, which gives the same value of T_4 and μ_4 . All the other supersymmetric D4-brane configurations are again related to the above two cases by $SO(3) \times SO(4)$ rotations.

3.4 D6 brane

All supersymmetric configurations are equivalent to the D6-brane with world volume (1245678). Again

$$\mu_6 \int [A_6] = \mu_6 \int d^6 x a_3 \quad (3.16)$$

and we can borrow the computational result from the D0-brane. The other source terms are

$$\begin{aligned} \mathcal{L}'_{\text{source}} &= \frac{iT_6}{2} \left(\sum_{i=1245678} h_{ii} + \frac{3}{2}\phi \right) \\ &= \frac{iT_6}{2} \left(\sum_{i=125678} h_{ii}^\perp + \frac{1}{2}\phi_0 + \frac{3}{8}(\phi_2 + \bar{\phi}_2) + \frac{1}{8}(\phi_6 + \bar{\phi}_6) \right). \end{aligned} \quad (3.17)$$

By the similar calculation, one can obtain the interaction energy

$$E = -\kappa^2 I_0^3 \left[T_6^2 + \frac{T_6^2}{2} \cos \mu r^+ + \left(\frac{T_6^2}{2} \mp \mu_6^2 \right) \cos \frac{\mu r^+}{3} \mp \mu_6^2 \cos \frac{2\mu r^+}{3} \right]. \quad (3.18)$$

Thus

$$T_6^2 = \mu_6^2 = \frac{\pi}{\kappa^2} (4\pi^2 \alpha')^{-3} \quad (3.19)$$

to match the string theory computation. In conclusion, we find that for all supersymmetric Dp-branes of interest their tensions and charges are given by

$$T_p^2 = \mu_p^2 = \frac{\pi}{\kappa^2} (4\pi^2 \alpha')^{3-p}. \quad (3.20)$$

4 Discussion of integrated amplitudes

In [19], the detailed study of interaction energy as functions of the brane separation in the IIB wave was presented. This is the continuation of their work on the branes and strings on the Type IIB plane wave background and AdS space[20, 21, 22]. Also see a related work[23]. Many special properties of the amplitude are traced to geometric properties of the plane wave background with emphasis on the lack of translational invariance. For special values of r^+ , the translational invariance is restored, which is related to the focusing of the geodesics. In the IIA case, many of the discussions can be carried over. Many features of the amplitude could again be traced to the lack of translational invariance and for special values of r^+ , the translational invariance is restored. In contrast to the IIB case, for some other special values of r^+ , the translational invariance is partially restored. This is related to the fact that the worldsheet theory of the Type IIA plane wave background consists of two (4,4) supermultiplets, one has mass $\frac{m}{3}$ and the other has mass $\frac{m}{6}$ while Type IIB theory correspond to two (4,4) supermultiplets with the same mass m , which leads to the (8,8) worldsheet supersymmetry.

In the Type IIA plane wave, the translation in the $x^I = (x^i, x^{i'})$ directions acts on

$$\begin{aligned} \delta r^- &= -\sin \frac{\mu}{3} x^+ \epsilon^i x^i - \sin \frac{\mu}{6} x^+ \epsilon^{i'} x^{i'} \\ \delta x^i &= \cos \frac{\mu}{3} x^+ \epsilon^i \\ \delta x^{i'} &= \cos \frac{\mu}{6} x^+ \epsilon^{i'} \end{aligned} \quad (4.1)$$

where i runs from 1 to 4 and i' runs from 5 to 8. This implies that the system of branes separated along the lightlike directions is not geometrically invariant under the translations in the x^I directions. The integrated propagators have very different behaviors than that of the flat case. The massless propagator $G_0(x_1, x_2)$ satisfying $\square G_0(x_1, x_2) = i\delta(x_1 - x_2)$ is given by

$$G_0(x_1, x_2) = i \left(\frac{\frac{\mu r^+}{3}}{\sin \frac{\mu r^+}{3}} \right)^2 \left(\frac{\frac{\mu r^+}{6}}{\sin \frac{\mu r^+}{6}} \right)^2 \int_0^\infty \frac{ds}{(4\pi i s)^5} \exp\left(-\frac{\sigma + i\epsilon}{4is}\right) \quad (4.2)$$

where

$$\begin{aligned}\sigma(x^i, x^{i'}) &= 2r^+ r^- + \frac{\frac{\mu r^+}{3}}{\sin \frac{\mu r^+}{3}} \Sigma_{i=1}^4 [(x_1^i x_1^i + x_2^i x_2^i) \cos \frac{\mu r^+}{3} - 2x_1^i x_2^i] \\ &+ \frac{\frac{\mu r^+}{6}}{\sin \frac{\mu r^+}{6}} \Sigma_{i'=5}^8 [(x_1^{i'} x_1^{i'} + x_2^{i'} x_2^{i'}) \cos \frac{\mu r^+}{6} - 2x_1^{i'} x_2^{i'}].\end{aligned}\quad (4.3)$$

Compared with the Type IIB case we observe the splitting of

$$\left(\frac{\mu r^+}{\sin \mu r^+}\right)^4 \quad (4.4)$$

into

$$\left(\frac{\frac{\mu r^+}{3}}{\sin \frac{\mu r^+}{3}}\right)^2 \left(\frac{\frac{\mu r^+}{6}}{\sin \frac{\mu r^+}{6}}\right)^2 \quad (4.5)$$

in the corresponding expression in the Type IIB case. This is related to the fact that the 8 transverse components of Type IIB theory involves the harmonic oscillator eigenfunctions with the parameter μr^+ , while in Type IIA theory 4 components have those with $\frac{\mu r^+}{3}$ and the other components have those with $\frac{\mu r^+}{6}$. If we take the flat space limit $\mu \rightarrow 0$ we have a translational invariance along x^i and $x^{i'}$ directions and $G_0(x_1, x_2)$ of (4.2) is reduced to the propagator in the flat space. If we integrate the $p+1$ dimensional world volume directions in the $\mu \rightarrow 0$ limit, we obtain the amplitude in the flat space

$$Z_{flat} \sim V_{p+1} (\sigma_{(9-p)}^2)^{-\frac{7-p}{2}} \quad (4.6)$$

but while in the IIA case

$$Z_p \sim \int_0^\infty \frac{ds}{s^{4-p}} \exp(-\frac{\sigma_{9-p}}{is}) \quad (4.7)$$

which gives the brane/brane exchange behavior of (2.13). We don't have any volume factors and the power law of the amplitude in the flat space is modified.

This behavior also can be seen from that of supergravity fields. The long range supergravity field sourced by the brane depends on the position of the Neumann directions. If we look for the behavior of a massless supergravity mode Ψ ($c=0$) at a given point far from the brane, one gets

$$\Psi \sim \left(\frac{\frac{\mu r^+}{3}}{\sin \frac{\mu r^+}{3}}\right)^2 \left(\frac{\frac{\mu r^+}{6}}{\sin \frac{\mu r^+}{6}}\right)^2 \tan^{\frac{n_1}{2}} \frac{\mu r^+}{3} \tan^{\frac{n_2}{2}} \frac{\mu r^+}{6} \tilde{\sigma}^{n_1+n_2-8} \quad (4.8)$$

where n_1 and n_2 is the number of Neumann directions in x^i directions and in $x^{i'}$ directions respectively with $n_1 + n_2 = p+1$ and

$$\tilde{\sigma} = -\frac{\mu r^+}{3} \tan \frac{\mu r^+}{3} \Sigma_{r=1}^{n_1} (x^r)^2 - \frac{\mu r^+}{6} \tan \frac{\mu r^+}{6} \Sigma_{r'=5}^{n_2+4} (x^{r'})^2 + \sigma(x^r=0, x^{r'}=0). \quad (4.9)$$

As happens in the IIB case, the powerlaw dependence is the same as in the flat space but the field sourced is not translationally invariant along the directions parallel to the brane. Integrating with respect to $x^r, x^{r'}$ of Neumann directions gives the brane/brane exchange behavior.

However, for special values of the separation, the translational invariance is restored. If $\mu r^+ = 6n\pi$ with n being an integer, the translation action (4.1) is given by

$$\delta x^- = 0, \quad \delta x^i = \epsilon^i \quad \delta x^{i'} = (-1)^n \epsilon^{i'} \quad (4.10)$$

while if $\mu r^+ = 3(2n+1)\pi$

$$\delta x^- = \frac{\mu}{6} (-1)^{n+1} \epsilon^{i'} x^{i'} \quad \delta x^i = -\epsilon^i \quad \delta x^{i'} = 0. \quad (4.11)$$

In this case we have the translational invariance along x^i directions but not along $x^{i'}$ directions. Depending on the value of μr^+ one can have the full restoration of translational invariance or the partial restoration of the translational invariance. The special value of μr^+ could be understood from the behavior of the geodesics as well. The geodesics in the Type IIA plane wave background is given by

$$r^- = \tilde{c}r^+ + \frac{\mu}{3} \sum_{i=1}^4 \left[\frac{1}{4} ((p_0^i)^2 - (x_0^i)^2) \sin \frac{2\mu r^+}{3} + \frac{1}{2} x_0^i p_0^i \cos \frac{2\mu r^+}{3} \right] \quad (4.12)$$

$$\begin{aligned} & + \frac{\mu}{6} \sum_{i'=5}^8 \left[\frac{1}{4} ((p_0^{i'})^2 - (x_0^{i'})^2) \sin \frac{\mu r^+}{3} + \frac{1}{2} x_0^{i'} p_0^{i'} \cos \frac{\mu r^+}{3} \right] \\ x^i &= x_0^i \cos \frac{\mu r^+}{3} + p_0^i \sin \frac{\mu r^+}{3} \\ x^{i'} &= x_0^{i'} \cos \frac{\mu r^+}{6} + p_0^{i'} \sin \frac{\mu r^+}{6} \end{aligned} \quad (4.13)$$

if we parametrize the geodesics by r^+ . A generic geodesic will reconverge to its original transverse position after $\mu r^+ = 12n\pi$. Note that for $\mu r^+ = 6n\pi$

$$x^{i'} = (-1)^n x_0^{i'}, \quad x^i = x_0^i. \quad (4.14)$$

For $\mu r^+ = 6n\pi$ the field theory result has the reinstated translational invariance. If $x_1^i \neq x_2^i$ and $x_1^{i'} \neq (-1)^n x_2^{i'}$, the geodesics will be of infinite distance and we regularize the geodesic distance

$$\mu r^+ = 6n\pi + \epsilon \quad (4.15)$$

then

$$\sigma_{10} = \left(\frac{6n\pi}{\epsilon} + 1 \right) [\sum_i (x_1^i - x_2^i)^2 + \sum_{i'} (x_1^{i'} - (-1)^n x_2^{i'})^2] + 2r^+ r^- + o(\epsilon). \quad (4.16)$$

This expression for the distance is very similar to that of the flat space. In this limit the field theory exchange expression is given by

$$\begin{aligned} Z_p &\sim V_{n_1} V_{n_2} \epsilon^{\frac{n_1+n_2}{2}} \int \frac{ds}{s^{5-\frac{n_1}{2}-\frac{n_2}{2}}} \exp \frac{i\sigma_{(10-n_1-n_2)}}{s} \\ &\sim \frac{V_{n_1} V_{n_2} \epsilon^4}{[\sum_{r=n_1+1}^4 (x_1^r - x_2^r)^2 + \sum_{r'=n_2+5}^8 (x_1^{r'} - (-1)^n x_2^{r'})^2]^{4-\frac{n_1}{2}-\frac{n_2}{2}}} \end{aligned} \quad (4.17)$$

, which shows the reinstated translational invariance.

If $x_1^r = x_2^r$ for j_1 directions among x^i 's and $x_1^{r'} = (-1)^n x_2^{r'}$ for j_2 directions among $x^{i'}$'s then Z_P is given by

$$Z_P \sim \frac{V_{n_1} V_{n_2} \hat{V}_{j_1} \hat{V}_{j_2} \epsilon^4}{[\sum_{r=j_1+1}^4 (x_1^r - x_2^r)^2 + \sum_{r'=j_2+5}^8 (x_1^{r'} - (-1)^n x_2^{r'})^2]^{4 - \frac{n_1+n_2}{2} - \frac{j_1+j_2}{2}}} \quad (4.18)$$

Here \hat{V}_j is the regulated momentum space volume for the j_1, j_2 directions.¹ If $\mu r^+ = 3(2n+1)\pi + \epsilon$

$$\sigma_{10} = 2r^+ r^- - \frac{3(2n+1)\pi}{\epsilon} [\sum_{i=1}^4 (x_1^i + x_2^i)^2] + (-1)^{n+1} \frac{(2n+1)\pi}{2} [\sum_{i'=5}^8 2x_1^{i'} x_2^{i'}] + o(\epsilon) \quad (4.19)$$

The translational invariance is restored only in the x^i directions. If $x_1^i \neq -x_2^i$

$$Z_P \sim V_{n_1} \epsilon^{\frac{n_1}{2}} \int \frac{ds}{s^{5-\frac{n_1}{2}}} \exp \frac{i\sigma(10-n_1-n_2)}{s} \quad (4.20)$$

$$\sim \frac{V_{n_1} \epsilon^4}{[\sum_{r=n_1+1}^4 (x_1^r - x_2^r)^2]^{4 - \frac{n_1}{2}}}. \quad (4.21)$$

If $x_1^i = -x_2^i$ for j_1 directions

$$Z_P \sim \frac{V_{n_1} \hat{V}_{j_1} \epsilon^4}{\sum_{i=n_1+j_1+1}^4 ((x_1^i + x_2^i)^2)^{4 - \frac{n_1+j_1}{2}}} \quad (4.22)$$

and if $x_1^i = -x_2^i$ for all i we have

$$Z_P \sim V_{n_1} \hat{V}_{4-n_1} \epsilon^2 \int \frac{ds}{s^3} \exp \frac{i}{s} (2r^+ r^- + (-1)^{n+1} \frac{(2n+1)\pi}{2} \sum_{i'=n_1+5}^8 2x_1^{i'} x_2^{i'}) \quad (4.23)$$

In all the cases where the translation invariance is at least partially restored, the amplitude vanishes in the limit $\epsilon \rightarrow 0$, whose feature is shared by the exchange amplitudes of the supersymmetric D-branes in the flat space.

As happens in the Type IIB case, the special features are also observed in the worldsheet calculation. We follow the convention in [3, 4, 7]. If $\mu r^+ \rightarrow 3\pi$, the bosonic mode expansion in X^i directions develop new zero modes

$$\begin{aligned} X^{iN} &= (x^{iN} + \alpha' p^{iN} \tau) 2 \cos \sigma + \dots \\ X^{iD} &= x_0^{iD} \cos \sigma + (x^{iD} + \alpha' p^{iD} \tau) 2 \sin \sigma + \dots \end{aligned} \quad (4.24)$$

with $[x^i, p^j] = i\delta^{ij}$. The corresponding fermionic zero modes are given by

$$\begin{aligned} \Psi_2^+ &= \sqrt{\frac{\alpha'}{2}} (\Psi_1 e^{-i\sigma} - i\gamma^4 \Omega \Psi_1 e^{i\sigma} + \Psi_{-1} e^{i\sigma} + i\gamma_4 \Omega \Psi_{-1} e^{-i\sigma}) + \dots \\ \Psi_1^- &= \sqrt{\frac{\alpha'}{2}} (\Omega \Psi_1 e^{i\sigma} - i\gamma^4 \Psi_1 e^{-i\sigma} + \Omega \Psi_{-1} e^{-i\sigma} + i\gamma_4 \Psi_{-1} e^{i\sigma}) + \dots \end{aligned} \quad (4.25)$$

¹We use $\delta(x) = \lim_{\alpha \rightarrow \infty} \frac{\alpha}{\sqrt{\pi}} \exp(-\alpha^2 x^2) = \frac{\hat{V}}{2\pi}$ with \hat{V} being the volume of the momentum space.

with $\{\Psi_1^a, \Psi_{-1}^b\} = \delta^{ab}$ where a, b runs from 1 to 4. Ω is the matrix relating fermionic left and right modes in the open string sectors whose detailed forms are dependent on the D-branes of interest and are given in [2, 7]. No new zero bosonic modes in $X^{i'}$ directions appear and neither the fermionic modes corresponding to their superpartners do. The interaction energy for the brane/brane in this case is given by

$$E = V_{n_1} V_{4-n_1} \int_0^\infty \frac{dt}{t} \exp\left(\frac{3ir^-}{\alpha'\mu} t\right) (2\sinh\pi t)^{2-n_1} \left(2\sinh\frac{\pi t}{2}\right)^{2-n_2} (1-1)^4 \quad (4.26)$$

where V_{n_1} is the volume of the Neumann directions within x^i directions and V_{4-n_1} is the volume of the remaining Dirichlet directions in the x^i directions and $(1-1)^4$ comes from the fermionic zero modes, which annihilates the amplitude. Existence of continuous modes in the Neumann directions reflects the partially reinstated translational invariance along the world volume directions within x^i directions leading to V_{n_1} factors. There are also continuous modes in the Dirichlet directions leading to V_{4-n_1} factor. This follows from the infinite family of geodesics connecting x_0^{iD} to $-x_0^{iD}$.

If $\mu r^+ \rightarrow 6\pi$ all of the bosonic and the fermionic modes develop new zero modes. The bosonic zero modes are given by

$$\begin{aligned} X^{i'_N} &= (x^{i'_N} + \alpha' p^{i'_N} \tau) 2\cos\sigma + \dots \\ X^{i'_D} &= x_0^{i'_D} \cos\sigma + (x^{i'_D} + \alpha' p^{i'_D} \tau) 2\sin\sigma + \dots \end{aligned} \quad (4.27)$$

and the fermionic zero modes are given by

$$\begin{aligned} \Psi_2^- &= \sqrt{\frac{\alpha'}{2}} (\Psi'_1 e^{-i\sigma} + i\gamma^4 \Omega \Psi'_1 e^{i\sigma} + \Psi'_{-1} e^{i\sigma} - i\gamma_4 \Omega \Psi'_{-1} e^{-i\sigma}) + \dots \\ \Psi_1^+ &= \sqrt{\frac{\alpha'}{2}} (\Omega \Psi'_1 e^{i\sigma} + i\gamma^4 \Psi'_1 e^{-i\sigma} + \Omega \Psi'_{-1} e^{-i\sigma} - i\gamma_4 \Omega \Psi'_{-1} e^{i\sigma}) + \dots \end{aligned} \quad (4.28)$$

while for x^i directions we just replace σ by 2σ for the bosonic zero modes and the fermionic zero modes in (4.27) and (4.28). The interaction energy is given by

$$E = V_{n_1} V_{4-n_1} V_{n_2} V_{4-n_2} \int_0^\infty \frac{dt}{t} \exp\left(\frac{6ir^-}{\alpha'\mu} t\right) (2\sinh 2\pi t)^{2-n_1} (2\sinh\pi t)^{2-n_2} (1-1)^8 \quad (4.29)$$

where $(1-1)^8$ part comes from the fermionic zero modes, V_{n_2} is the volume of the Neumann directions within $x^{i'}$ directions and V_{4-n_2} is the volume of the remaining Dirichlet directions within $x^{i'}$ directions.

A Appendix

In this appendix, we explain how to obtain the bulk action of the relevant field for the computation in the main text. In [18], the equations of motion for the IIA supergravity

fluctuation to the leading order are obtained. They consists of the decoupled equation of motion of the form

$$\square\phi_c + i\frac{1}{3}\mu c\partial_-\phi_c = 0 \quad (\text{A.1})$$

We use the leftmost subscript to indicate the coefficient c in the above equation in writing the various fields. The bosonic fields of IIA supergravity theory consist of the dilaton Φ , graviton $g_{\mu\nu}$, one-form field A_μ , two form field $B_{\mu\nu}$ and three form field potential $A_{\mu\nu\rho}$. We consider the small fluctuation of the above fields on the IIA plane wave background.

$$\begin{aligned} \Phi &= \phi & g_{\mu\nu} &= \bar{g}_{\mu\nu} + h_{\mu\nu} \\ A_\mu &= \bar{A}_\mu + a_\mu & B_{\mu\nu} &= b_{\mu\nu} \\ A_{\mu\nu\rho} &= \bar{A}_{\mu\nu\rho} + a_{\mu\nu\rho} \end{aligned} \quad (\text{A.2})$$

where $\bar{g}_{\mu\nu}$, \bar{A}_μ and $\bar{A}_{\mu\nu\rho}$ denote the IIA plane-wave background. We take the light cone gauge.

$$a_- = b_{-I} = a_{-IJ} = h_{-I} = 0 \quad (\text{A.3})$$

Since the IIA plane-wave background has $SO(3) \times SO(4)$ symmetry, it's convenient to classify the small fluctuations using this symmetry. $SO(3) \times SO(4)$ scalar multiplets are given by

$$\begin{aligned} \phi_0 &\equiv \phi + \frac{1}{3}h_{ii} + h_{44} , \\ \phi_2 &\equiv \phi + \frac{4}{3}ia_4 - \frac{2}{3}h_{44} , & \bar{\phi}_2 &\equiv \phi - \frac{4}{3}ia_4 - \frac{2}{3}h_{44} , \\ \phi_6 &\equiv \phi - 4ia_{123} - 2h_{ii} , & \bar{\phi}_6 &\equiv \phi + 4ia_{123} - 2h_{ii} . \end{aligned} \quad (\text{A.4})$$

Here the subscript indicates the coefficient appearing in the mass parameter in the equation of motion (A.1). In the appendix, i denotes $SO(3)$ direction while i' denotes $SO(4)$ one. For example, ϕ_6 satisfies

$$\square\phi_6 + i2\mu\partial_-\phi_6 = 0 \quad (\text{A.5})$$

and its complex conjugate $\bar{\phi}_6$ satisfies

$$\square\bar{\phi}_6 - i2\mu\partial_-\bar{\phi}_6 = 0. \quad (\text{A.6})$$

$SO(3)$ vector multiplets are

$$\begin{aligned} \beta_{0i} &\equiv b_{4i} \\ \beta_{2i} &\equiv \sqrt{2}a_i - ih_{4i} + \frac{i}{2}\epsilon_{ijk}b_{jk} + \sqrt{2}\epsilon_{ijk}a_{4jk}, & \bar{\beta}_{2i} &\equiv \sqrt{2}a_i + ih_{4i} - \frac{i}{2}\epsilon_{ijk}b_{jk} + \sqrt{2}\epsilon_{ijk}a_{4jk}, \\ \beta_{4i} &\equiv \sqrt{2}a_i + ih_{4i} + \frac{i}{2}\epsilon_{ijk}b_{jk} - \sqrt{2}\epsilon_{ijk}a_{4jk}, & \bar{\beta}_{4i} &\equiv \sqrt{2}a_i - ih_{4i} - \frac{i}{2}\epsilon_{ijk}b_{jk} - \sqrt{2}\epsilon_{ijk}a_{4jk} \end{aligned} \quad (\text{A.7})$$

Here again the leftmost subscript denotes the coefficient in the mass parameter. $SO(4)$ vector multiplets are given by

$$\begin{aligned}\beta_{1i'} &\equiv \sqrt{2}a_{i'} + ih_{4i'}, & \bar{\beta}_{1i'} &\equiv \sqrt{2}a_{i'} - ih_{4i'}, \\ \beta_{3i'} &\equiv b_{4i'} - \frac{i}{3}\epsilon_{i'j'k'l'}a_{j'k'l'}, & \bar{\beta}_{3i'} &\equiv b_{4i'} + \frac{i}{3}\epsilon_{i'j'k'l'}a_{j'k'l'}\end{aligned}\quad (\text{A.8})$$

Then there are two index tensor fields. (ij) and $(i'j')$ components are $SO(3)$ and $SO(4)$ graviton multiplets respectively. Thus we have

$$h_{ij}^\perp \equiv h_{ij} - \frac{1}{3}\delta_{ij}h_{kk}, \quad h_{i'j'}^\perp \equiv h_{i'j'} - \frac{1}{4}\delta_{i'j'}h_{k'k'} \quad (\text{A.9})$$

and the equations of motion are written as

$$\square h_{ij}^\perp = 0, \quad \square h_{i'j'}^\perp = 0 \quad (\text{A.10})$$

And (ij') components are defined by

$$\beta_{1ij'} \equiv b_{ij'} + i\sqrt{2}a_{4ij'}, \quad \bar{\beta}_{1ij'} \equiv b_{ij'} - i\sqrt{2}a_{4ij'}, \quad (\text{A.11})$$

and $(i'j')$ components are

$$\begin{aligned}\beta_{2i'j'} &\equiv a_{i'j'}^+ - i\tilde{a}_{i'j'}^+, & \bar{\beta}_{2i'j'} &\equiv a_{i'j'}^+ + i\tilde{a}_{i'j'}^+, \\ \beta_{4i'j'} &\equiv a_{i'j'}^- + i\tilde{a}_{i'j'}^-, & \bar{\beta}_{4i'j'} &\equiv a_{i'j'}^- - i\tilde{a}_{i'j'}^-, \end{aligned} \quad (\text{A.12})$$

where

$$a_{i'j'}^\pm \equiv b_{i'j'} \pm \frac{1}{2}\epsilon_{i'j'k'l'}b_{k'l'}, \quad \frac{1}{\sqrt{2}}\tilde{a}_{i'j'}^\pm \equiv a_{4i'j'} \pm \frac{1}{2}\epsilon_{i'j'k'l'}a_{4k'l'}.$$

Finally we have two types of 3-rank tensor fields. (ijk') components are

$$\beta_{3ijk'} \equiv \sqrt{\frac{2}{3}}a_{ijk'} - i\epsilon_{ijk}h_{kk'}, \quad \bar{\beta}_{3ijk'} \equiv \sqrt{\frac{2}{3}}a_{ijk'} + i\epsilon_{ijk}h_{kk'}, \quad (\text{A.13})$$

while $(ij'k')$ components are

$$\beta_{0ij'k'} \equiv a_{ij'k'}. \quad (\text{A.14})$$

Consider the above diagonalized fields. One can obtain the action which gives the equation of motion of each field. Since we started with the equation of motion, there are same ambiguities in the relative normalization of the fields. However such relative normalizations could be fixed by matching the gravity and the 2-form potential parts with the known form of the IIA actions in the light-cone gauge.

The relevant part of the action needed for the computation in the main text is given by

$$\begin{aligned}
\mathcal{L} = & \frac{1}{8\kappa^2} (h_{ij}^\perp \square h_{ij}^\perp + h_{i'j'}^\perp \square h_{i'j'}^\perp) \\
& + \frac{3}{32\kappa^2} \phi_0 \square \phi_0 + \frac{9}{64\kappa^2} \bar{\phi}_2 (\square + \frac{2i\mu}{3} \partial_-) \phi_2 + \frac{1}{64\kappa^2} \bar{\phi}_6 (\square + 2i\mu \partial_-) \phi_6 \\
& + \frac{1}{16\kappa^2} \bar{\beta}_{2i} (\square + \frac{2i\mu}{3} \partial_-) \beta_{2i} + \frac{1}{16\kappa^2} \bar{\beta}_{4i} (\square + \frac{4i\mu}{3} \partial_-) \beta_{4i} \\
& + \frac{1}{8\kappa^2} \bar{\beta}_{1i'} (\square + \frac{i\mu}{3} \partial_-) \beta_{1i'} + \frac{1}{16\kappa^2} \bar{\beta}_{3i'} (\square + i\mu \partial_-) \beta_{3i'} \\
& + \frac{1}{16\kappa^2} \bar{\beta}_{1ij'} (\square + \frac{i\mu}{3} \partial_-) \beta_{1ij'} \\
& + \frac{1}{64\kappa^2} \bar{\beta}_{2i'j'} (\square + \frac{2i\mu}{3} \partial_-) \beta_{2i'j'} + \frac{1}{64\kappa^2} \bar{\beta}_{4i'j'} (\square + \frac{4i\mu}{3} \partial_-) \beta_{4i'j'} \\
& + \frac{1}{16\kappa^2} \bar{\beta}_{3ijk'} (\square + i\mu \partial_-) \beta_{3ijk'} \\
& + \dots
\end{aligned} \tag{A.15}$$

where the omitted terms are not relevant for the computation in the main text. There are slight differences in the field definitions adopted here and in [18]. Our convention is that the coefficient in front of each field is consistent with the usual normalization of the p-form field

$$\frac{1}{p!} \frac{1}{4\kappa^2} \int d^{10}x A_{[i_1 \dots i_p]} \square A_{[i_1 \dots i_p]} = \frac{1}{4\kappa^2} \int d^{10}x \sum_{i_1 < i_2 < \dots < i_p} A_{i_1 i_2 \dots i_p} \square A_{i_1 i_2 \dots i_p}. \tag{A.16}$$

The integrated propagator $I_c^{9-p}(r^+, r^-)$ associated with (A.1) is simply $e^{\frac{ic\mu r^+}{3}} I_0^{9-p}(r^+, r^-)$.

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